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DIFFERENTIAL EQUATIONS IN ECONOMIC PROBLEMS USING MATHEMATICAL MODELS

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Abstract: *This article examines the application of differential equations in economic problems, and also examines models of processes and demand functions using differential equations with separable and separated variables. The article shows how to make differential equations for solving economic problems and making their models. Also, the article shows examples of economic problems and their models.*

Key words: *First order differential equations with separable variables, demand function, investment rate, investment income, financial market models.*

INTRODUCTION

Differential equations are actively used to develop mathematical models of living systems. An important aspect of the application of differential equations is the construction of high-dimensional models that take into account the spatial organization of a particular living system.

Differential equations play an important role in economic models because they allow us to describe dynamic processes and changes over time. Here are some examples of their application in economics: DE is needed for a growth model. Differential equations are used to model economic growth, for example, capital and labor affect economic growth.

In Consumption and Saving Models, differential equations help analyze how consumption and saving change over time depending on income and other factors.

Also, DE is needed for estimating for modeling inflation processes and analyzing the impact of monetary policy on the price level. DE is needed to calculate the model of financial markets. Differential equations are used to describe the dynamics of prices in financial markets, including option pricing models.

LITERATURE ANALYSIS

Mathematical modeling is a powerful tool for researching and analyzing various phenomena and processes. It allows predicting results, optimizing solutions, and making more informed decisions in various fields of science. Also, mathematical modeling allows us to identify the most important properties of an object for research, abstracting from its insignificant characteristics. Modeling often allows us to formulate new hypotheses and obtain new knowledge about the object that was not available during its research. Mathematical models use: formulas, equations, inequalities, systems of equations, which make it possible to describe with some accuracy the phenomena and processes occurring in the original [1].

Economic problems are often solved using mathematical models that allow us to analyze and predict various economic processes. Here are some examples of such problems:

Differential equations are used in production optimization. Here, linear programming models help determine the optimal allocation of resources to maximize profits or minimize costs.

Differential equations are needed to analyze supply and demand. Regression models are used to predict demand for goods and services depending on various factors such as price, income, and competition.

Differential equations are needed for macroeconomic modeling. General equilibrium models allow us to analyze the impact of various economic policies on the economy as a whole, including changes in taxes, government spending, and monetary policy.

Differential equations are needed for financial modeling. Asset valuation and risk management models help investors make informed decisions in financial markets.

RESEARCH METHODOLOGY

Let us consider the application of first-order differential equations for constructing mathematical models in some economic problems: Model of natural growth of output.

A first-order differential equation with separable variables is called an equation of the form: $P(x)\,Q(y)\,dx + R(x)\,S(y)\,dy = 0$ [6].

Let some products be sold at a fixed price P.

Let Q(t) denote the quantity of products sold at time t; then at this time the income received is equal to PQ(t).

Let part of this income be spent on investments in the production of the products sold, i.e.

$$I(t)=mPQ(t) \quad (1),$$

where m is the investment rate - a constant number, and 0 < m < 1.

If we proceed from the assumption that the market is not saturated (or that the manufactured products are fully realized), then as a result of the expansion of production, an increase in income will be obtained, part of which will again be used to expand the output of products.

This will lead to an increase in the rate of output (acceleration), and the rate of output is proportional to the increase in investment, i.e.

$$Q'=lI \quad (2),$$

where $1/l$ is the rate of acceleration. Substituting formula (1) into (2), we obtain

$$Q'=kQ,\ k=lmP \quad (3)$$

Differential equation (3) is a first-order equation with separable variables [6]. The general solution to this equation is $Q=Ce^{kt}$, where C is an arbitrary constant. Let the output volume Q_0 be fixed (set) at the initial moment of time $t=t_0$. Then, from this condition, the constant C can be expressed: $Q_0 = Ce^{kt_0}$, whence $C = Q_0e^{-kt_0}$.

From here we obtain a particular solution to equation (3) - the solution to the Cauchy problem for this equation:

$$Q = Q_0 = Ce^{k(t-t_0)} \quad (4)$$

Therefore, with the help of a differential equation, it is possible to create mathematical models in many areas: for example, in the results of biological experiments, where the process of bacterial reproduction occurs; or in the process of radioactive decay, which obeys the law established by formula (4), in economic problems.

MAIN PART

Now, let us consider problems in which models of processes are created using differential equations with separable and separated variables.

In particular, for example, supply and demand are economic categories of commodity production that arise and function in the market, in the sphere of commodity exchange [2].

Let's take some product. Let's denote the price of the product by p, and let's denote the formation of the price (the derivative of the price over time) by $\frac{dp}{dt} = p'$. In this problem, supply and demand depend on the rate of price change. But the price of the product can also be formed using various functions. One of such economic laws of commodity production is the law of supply and demand, which consists in the interdependence of supply and demand and their objective desire to correspond.

For example, the demand and supply functions are: $s = 5\frac{dp}{dt} + p + 10$

$$q = 2\frac{dp}{dt} - p + 15 - e^{-2t}$$

Equilibrium between supply and demand is maintained under the condition that under the condition that

$$5\frac{dp}{dt} + p + 10 = 2\frac{dp}{dt} - p + 15 - e^{-2t}$$

or, when the equality holds

$$3\frac{dp}{dt} = -2p + 5 - e^{-2t}$$

As a result, a linear non-homogeneous differential equation of the first order with constant coefficients is obtained. To solve it, we apply the method of I. Bernoulli: we seek a general solution in the form p=uv, where u=u(t), v=v(t) [7]. Then p′=u′v+uv′. Substituting for p and p′, we obtain the equation:

$3p'+2p=5-e^{-2t}$, $u'v+uv'+2uv = 5-e^{-2t}$, $u'v+u(v'+2v) = 5-e^{-2t}$, $v'+2v=0$

$\frac{dv}{dt} = -2v$, $\int \frac{dv}{v} = -2\int dt$, $ln|v| = -2t$, $v = e^{-2t}$, $u'e^{-2t} = 5 - e^{-2t}$,

$u' = 5e^{2t} - 1$, $du = (5e^{2t} - 1)dt$, $u = \frac{5}{2}e^{2t} - t + C$

$p = uv = (\frac{5}{2}e^{2t} - t + C)e^{-2t}$, $p = (\frac{5}{2}e^{2t} - t + C)e^{-2t}$- General solution to the equation.

Let us find the dependence of the equilibrium price on time, if at the initial moment the price was p = 20, at t = 0, then:

$20 = (\frac{5}{2}e^{2\cdot 0} - 0 + C)e^{-2\cdot 0}$. C=8

Thus, the desired dependence has the form

$$p = \left(\frac{5}{2}e^{2t} - t + 8\right)e^{-2t} = (2{,}5e^{2t} - t + 8)e^{-2t}$$

To find out whether a given equilibrium price is stable, we find

$$\lim_{t\to\infty} p = \lim_{t\to\infty} \frac{2{,}5e^{2t} - t + 8}{e^{2t}}$$

in this case, the uncertainty ∞/∞ obtained during the calculations will be disclosed according to L'Hôpital's rule.

$$\lim_{t\to\infty} \frac{(2{,}5e^{2t} - t + 8)'}{(e^{2t})'} = \lim_{t\to\infty} \frac{5e^{2t} - 1}{2e^{2t}} = \lim_{t\to\infty} \frac{(5e^{2t} - 1)'}{(2e^{2t})'} = \lim_{t\to\infty} \frac{10e^{2t}}{4e^{2t}} = 2{,}5$$

Therefore, the equilibrium price is stable.

Now consider a model of a market with predictable prices.

This model uses the theory of linear differential equations of the second order. Usually, in simple models of a market economy, supply and demand depend on the current price of a product. However, in real situations there is a dependence on the pricing trend and the rate of price change. In models with continuous and time-differentiable t functions, these characteristics are described by the first and second derivatives of the price function p(t), respectively.

If p″>0, then the interest in the product on the market also grows and vice versa, if the price rate falls, then the interest in the product also falls. Moreover, a rapid price increase scares away the buyer, therefore the term with the first derivative of the price function is included with a minus sign. At the same time, the rate of price change affects the increase in supply, but the price increase increases supply, therefore the term containing p′ is included in q(t) with a plus sign.

Example. Let the demand functions s(t) and supply q(t) have the following dependencies on the price p: s(t)=p″-2p′-6p+36, q(t)=2p″+4p′+4p+6. Find the dependence of price on time.

Solution. Let's use the condition of the market equilibrium s(t) = q(t). Based on this condition, we obtain the equation: p″-2p′-6p+36=2p″+4p′+4p+6. Hence: p″+6p′+10p=30

This is a linear non-homogeneous differential equation of the second order with constant coefficients. Let us find the general solution of this differential equation. To do this, we will compose the characteristic equation: $k^2+6k+10=0$. $D=36-40 = -4$, $k_{1,2} = (-6\pm 2i)/2$, $k_1=-3-i$, $k_2=-3+i$,
General solution of the homogeneous equation: $p(t)= e^{-3t} (C_1 \cos t + C_2 \sin t)$
We will take a particular solution by the selection method: $\bar{p}(t) = At + B$. Тогда
$\bar{p}'(t) = A$, $\bar{p}''(t) = 0$. From here, substituting them into the equation $p''+6p'+10p=30$, we get: $6A+10(At+B) = 30$. From here:
$\begin{cases} x: & 10A = 0 \\ x^0: & 6A + 10B = 30 \end{cases}$ A=0, B=3. Therefore, the particular solution: $\bar{p}(t) = 3$.
General solution to the equation: $p(t)= e^{-3t} (C_1 \cos t + C_2 \sin t) + 3$
Considering that $\lim_{t\to\infty} p(t) = \lim_{t\to\infty} (e^{-3t}(C_1 cost + C_2 sint) + 3) = 3$,
Answer: all integral curves have a horizontal asymptote p = 3. This means that all prices tend to a stable price $\bar{p}(t)=3$ with fluctuations around it, and the amplitude of the fluctuations fades over time.

Example. Let the rate of increase in output of an enterprise be directly proportional to its profit with a proportionality coefficient k=1.2 and y(t) is the output of this enterprise. Create an equation linking the rate of change in output and the income from the sale of output at a price of p(y). It is assumed that with the increase in output the market will become saturated and the price of the goods will fall. It is known that the price of one unit of output is given by the function p(y)=5-2y. The full costs of the enterprise are expressed by the function c(y)=6y+2. Write a differential equation for the function y(t). Find the function given that at the initial moment of time the output is 200.

Solution: Let the output rate be y=y(t), where t is time, then the output rate will be: $\frac{dy}{dt}$, and let the enterprise revenue be defined as:

$y \cdot p(t)$ – is the product of the price y and the intensity of its output p(t). Then profit is defined as the difference between the enterprise's revenue and total costs c(y). The output rate is 1.1 times greater than its profit. It is necessary to find a function of the variable t.
We express all values through t and obtain the enterprise profit: $y \cdot p(t) - c(y) = y \cdot (5 - 3y) - (6y + 1) = -3y^2 - y - 1$
production speed: $y' = \frac{dy}{dt}$.
By composing a differential equation, we obtain: $y' = -1{,}1(3y^2 + y + 1)$
Let's find the general solution to the differential equation.
We obtained a first-order differential equation with separable variables [5].
By solving it we get the general solution: $y = \frac{e^{-\frac{11}{10}(t+c)}}{3(1-e^{-\frac{11}{10}(t+c)})}$
Now, substituting the initial condition y(0)=100, we find a particular solution.
$e^{-\frac{11}{10}(0+c)} = e^{-\frac{11}{10}c} = \frac{300}{301}$. Therefore, a particular solution satisfying the initial condition y(0)=100 has the form:

$$y = \frac{100e^{-\frac{11}{10}t}}{301 - 300e^{-\frac{11}{10}t}}$$

Now, examining the obtained solution,

t-time	0	0,258	0,5	1	2
y(t)-production output	100	1,002	0,449	0,163	0,041

The table shows that with increasing time the output intensity y(t) decreases.

Answer:

Thus, starting from the moment t=0.258 the output becomes less than one.

Now let us consider the following: let y=y(t) be the volume of production of some production, realized by the moment of time t. Let us assume that the price of this product remains constant (within the considered time interval). Then the function y=y(t) satisfies the condition y′=ky.

Let y=y(t) be the volume of production. Then the function y=y(t) satisfies the condition, $y'=ky$.

The equation is an equation with separable variables. Its solution has the form:

$y = y_0 e^{k(t-t_0)}$.

The equation describes such population growth, the dynamics of price growth with constant inflation, and further.

Example. Find the time interval in which the volume of sold products will double compared to the initial one if the value of the proportionality coefficient k in the equation is 0.1. By what % should the investment rate be increased so that the time interval required to double the volume of sold products decreases by 20%.

Solution: $t_0=0$, $k=0,1$, $y=2y_0 \Rightarrow 2y_0= y_0 e^{0,1t}$
$t=10\ln 2 \approx 6,93$ $t_1= 0,8t \Rightarrow k_1= k/0,8=1,25\, k \Rightarrow$

Answer: the investment rate should be increased by 25%.

The assumption of price constancy in practice turns out to be true only for narrow time intervals.

p is a decreasing function of the volume y of sold products

$p=p(y) \Rightarrow y' = mlp(y)y$

The last equation of the type also describes population growth in the presence of restrictions on this growth and the dynamics of epidemics.

RESULTS AND CONCLUSIONS

Differential equations are used in economics to model economic growth, gross domestic product, consumption, income and investment, while in finance stochastic differential equations are indispensable in modeling asset price dynamics and option pricing.

Equations are used in any field, they are attempts to describe real-world phenomena, including economics, as a mathematical model. No model is perfect, but some are useful. They can be used to explain theories from supply and demand to the marginal economics of taxation and comparative advantage in production.

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